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# On the Thermodynamics of Chiral Symmetry Restoration

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**Abstract**

The formulas for the temperature dependences of the non-strange and strange quark condensates are derived by taking into account the contribution of the massive resonances. Critical temperature of the chiral symmetry restoration transition is established to be 190 MeV if only meson resonances are considered, and 175 MeV if both meson and baryon resonances are taken into account.

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For massless quarks, the QCD Hamiltonian is invariant under chiral transformations. The phenomenology of the strong interactions is consistent with this property provided that chiral symmetry is spontaneously broken. Lattice gauge calculations [1] indicate that, at low temperatures, the scalar currents $\bar{q}q = \{\bar{u}u, \bar{d}d, \bar{s}s\}$ develop nonzero expectation values $\langle \bar{q}q \rangle_T$ which are referred to as quark condensates, and represent order parameter of spontaneously broken chiral symmetry. Chiral perturbation theory [2, 3, 4] shows that $\langle \bar{q}q \rangle_T$ melts as the temperature increases. One generally expects that if the temperature reaches a critical value, $T_c$, chiral symmetry is restored. Lattice calculations [5] confirm this expectation but show that the strange quark condensate $\langle \bar{s}s \rangle$ does not melt in the chiral restoration transition, supporting the description of this phase transition in terms of the behavior of the $SU(2) \times SU(2)$ linear $\sigma$-model initiated by Wilczek *et al.* [6]. Presumably, deconfinement transition which liberates color, takes place at temperature of the order of $T_c$. At low enough temperatures, chiral symmetry provides constraints on the temperature dependences of several physical quantities. For example, for the pion gas without interactions, one finds [3]

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{N_f^2 - 1}{3N_f} \frac{T^2}{4f_\pi^2}, \quad \frac{f_\pi(T)}{f_\pi} = 1 - \frac{N_f}{6} \frac{T^2}{4f_\pi^2}, \quad \frac{\mu_\pi(T)}{m_\pi} = 1 + \frac{1}{6N_f} \frac{T^2}{4f_\pi^2}, \quad (1)$$

where $f_\pi$ is the pion decay constant, $\simeq 93$ MeV, $\mu_\pi$ the screening pion mass, and $N_f$ the number of massless flavors. In a real world, $N_f = 2$, so that the first relation of (1) may be rewritten as

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} = 1 - \frac{T^2}{8f_\pi^2} \simeq 1 - \left( \frac{T}{260 \text{ MeV}} \right)^2. \quad (2)$$

If one goes beyond the low temperature limit, predictions start to become model dependent.

In this paper we calculate the correction to the formula for the non-strange quark condensate, Eq. (2), given by the contribution of the higher mass hadrons, and obtain a similar relation for the strange quark condensate. It is well known that the correct thermodynamic description of hot hadronic matter requires consideration of higher mass excited states, the resonances, whose contribution becomes essential at temperatures $\sim O(100 \text{ MeV})$ [7, 8, 9]. The method for taking into account these resonances was suggested by Belenky and Landau [10] as considering unstable particles

2

on an equal footing with the stable ones in the thermodynamic quantities; e.g., the formulas for the pressure and energy density in a resonance gas read[1]

$$p = \sum_i p_i = \sum_i g_i \frac{m_i^2 T^2}{2\pi^2} \sum_{r=1}^{\infty} (\pm 1)^{r+1} \frac{K_2(rm_i/T)}{r^2}, \tag{3}$$

$$\rho = \sum_i \rho_i, \quad \rho_i = T\frac{dp_i}{dT} - p_i, \tag{4}$$

where $+1$ $(-1)$ corresponds to the Bose-Einstein (Fermi-Dirac) statistics, and $g_i$ are the corresponding degeneracies ($J$ and $I$ are spin and isospin, respectively),

$$g_i = \begin{bmatrix} (2J_i + 1)(2I_i + 1) & \text{for non} - \text{strange mesons} \\ 4(2J_i + 1) & \text{for strange } (K) \text{ mesons} \\ 2(2J_i + 1)(2I_i + 1) & \text{for baryons,} \end{bmatrix}$$

and since chiral symmetry suppresses the interactions of low energy Goldstone bosons both among themselves and with massive hadrons, the gas can be approximately described as a collection of free particles.

To derive an explicit relation for the value of the quark condensate as a function of temperature, we first note that the operator $\bar{q}q$ occurs in the quark mass term of the Hamiltonian,

$$H = H_0 + \int d^3x \sum_{q=u,d,s} m_q\, \bar{q}q, \tag{5}$$

where $H_0$ is the Hamiltonian of massless (chirally symmetric) QCD. The thermal expectation value of $\bar{q}q$ represents, therefore, the response of the partition function to a change in the quark mass,

$$\langle\bar{q}q\rangle_T = -\frac{1}{V}\frac{\partial \ln Z}{\partial m_q}, \tag{6}$$

where $V$ is the four-dimensional euclidean volume $(= V^{(3)}/T)$. At sufficiently large volume, $\ln Z \to V(p - \rho_0)$, where $p$ is the pressure and $\rho_0$ the vacuum energy density. In the large volume limit, Eq. (6) therefore takes on the form

$$\langle\bar{q}q\rangle_T = \frac{\partial \rho_0}{\partial m_q} - \frac{\partial p}{\partial m_q} = \langle\bar{q}q\rangle_0 - \frac{\partial p}{\partial m_q}. \tag{7}$$

Since $p$ depends on $m_q$ only through the masses of the particles, one obtains from (3),(7), through $d/dx\,(x^2 K_2(x)) = -x^2 K_1(x)$,

$$\langle\bar{q}q\rangle_T = \langle\bar{q}q\rangle_0 + \sum_i g_i \frac{m_i^2 T}{2\pi^2}\frac{\partial m_i}{\partial m_q}\sum_r (\pm 1)^{r+1}\frac{K_1(rm_i/T)}{r}. \tag{8}$$

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[1]Since the temperatures we are dealing with are much less than the nucleon mass, we may treat all the particles as bosons, and distinguish fermions only by the factor 7/8 in the expression for the particle degeneracy. We also neglect chemical potential for simplicity.

1) Non-strange quark condensate.

One uses the lowest order relation [11]

$$m_\pi^2 f_\pi^2 = -m\langle \bar{q}q \rangle_0, \tag{9}$$

where $\langle \bar{q}q \rangle_0 \equiv \langle \bar{u}u + \bar{d}d \rangle_0$ and $m \equiv 1/2\,(m_u + m_d)$, and obtains from (8),

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle_0 \left( 1 - \frac{T^2}{f_\pi^2} \sum_i \frac{g_i}{2\pi^2} \frac{m_i m}{m_\pi^2} \frac{\partial m_i}{\partial m} \sum_r (\pm 1)^{r+1} \frac{m_i}{rT} K_1 \left( \frac{rm_i}{T} \right) \right). \tag{10}$$

1) Strange quark condensate.

Now one uses the lowest order relation for the $K$ meson ($f_K$ being the kaon decay constant, $\simeq 114$ MeV),

$$m_K^2 f_K^2 = -1/2\,(m + m_s)\langle \bar{s}s + 1/2\,\bar{q}q \rangle_0 \cong -m_s\langle \bar{s}s \rangle_0, \tag{11}$$

since $m_s \gg m$, and $\langle \bar{s}s \rangle_0 \simeq \langle \bar{u}u \rangle_0 \simeq \langle \bar{d}d \rangle_0$ [12], and obtains from (8),

$$\langle \bar{s}s \rangle_T = \langle \bar{s}s \rangle_0 \left( 1 - \frac{T^2}{f_K^2} \sum_i \frac{g_i}{2\pi^2} \frac{m_i m_s}{m_K^2} \frac{\partial m_i}{\partial m_s} \sum_r (\pm 1)^{r+1} \frac{m_i}{rT} K_1 \left( \frac{rm_i}{T} \right) \right). \tag{12}$$

Gerber and Leutwyler [4] have established the $T$-dependence of the non-strange quark condensate by direct calculation of the sum in Eq. (10) for known resonances, using the estimate $\partial m_i / \partial m \simeq N_i$, the number of valence quarks of type $u$ and $d$, and $m \simeq 7$ MeV. The results of Gerber and Leutwyler predict chiral symmetry to be restored at $T_c \simeq 200$ MeV if only massive states are taken into account, and with the pion contribution in addition to that of the massive states, $T_c \simeq 190$ MeV for nonzero masses of the $u$- and $d$-quarks, and 170 MeV for zero masses of the latter. In this paper we wish to obtain analytic expressions for both the $T$-dependent non-strange and strange quark condensates, and compare the results with those of Gerber and Leutwyler.

In the following we shall restrict ourselves to the meson resonances alone. Baryon resonances may be treated in a way similar to that described in this paper, with the introduction of two chemical potentials, for both conserved net baryon number and strangeness. We shall dwell briefly on this point at the end of the paper.

To calculate the derivatives $\partial m_i / \partial m_q$ in Eqs. (10),(12), we first note that the expansion of $m_\pi^2$ and $m_K^2$ in the powers of $m_u, m_d, m_s$ starts with terms which are linear in the quark masses,

$$m_\pi^2 = 2mB, \quad m_K^2 = (m + m_s)B. \tag{13}$$

For higher mass meson resonances, we expect that similar relations hold (with $C$ being constant within a given meson nonet):

$$m_{0''}^2 \simeq m_1^2 = 2mB + C, \quad m_{1/2}^2 = (m + m_s)B + C, \quad m_{0'}^2 \simeq 2m_s B + C, \tag{14}$$

4

where $m_1, m_{1/2}, m_{0'}, m_{0''}$ are the masses of the isovector, isospinor and two isoscalar states, respectively, and $0'$ belongs to a mostly octet. Indeed, for the vector meson nonet these relations may be easily obtained from the constancy of the differences of the squared masses of the corresponding spin-triplet and spin-singlet states [13],

$$\triangle M^2 \equiv M^2(^3S_1) - M^2(^1S_0),$$

resulting in[2]

$$m_\rho^2 - m_\pi^2 = 0.57 \text{ GeV}^2, \quad m_{K^*}^2 - m_K^2 = 0.55 \text{ GeV}^2, \tag{15}$$

and two Gell-Mann–Okubo mass formulas, the standard one [14],

$$m_1^2 + 3m_8^2 = 4m_{1/2}^2, \tag{16}$$

and an extra relation [15, 16]

$$m_{0'}^2 + m_{0''}^2 = m_8^2 + m_0^2 = 2m_{1/2}^2, \tag{17}$$

with $m_0$ and $m_8$ being the masses of the isoscalar octet and singlet states, respectively, which for an almost ideally mixed nonet reduces to [15, 16]

$$m_{0''}^2 \simeq m_1^2, \quad m_{0'}^2 \simeq 2m_{1/2}^2 - m_1^2. \tag{18}$$

Therefore, it follows from (13),(15),(18) that

$$m_\omega^2 \simeq m_\rho^2 = 2mB + C, \quad m_{K^*}^2 = (m + m_s)B + C, \quad m_\phi^2 \simeq 2m_sB + C,$$

with $C \simeq 0.56 \text{ GeV}^2$. It was shown by Balázs [17] that $m_\rho^2 - m_\pi^2 = 1/2\alpha'$, with $\alpha'$ being a universal Regge slope, $\alpha' \simeq 0.85 \text{ GeV}^{-2}$, in agreement with (15), so that

$$m_\omega^2 \simeq m_\rho^2 = 2mB + 1/2\alpha', \quad m_{K^*}^2 = (m + m_s)B + 1/2\alpha', \quad m_\phi^2 \simeq 2m_sB + 1/2\alpha'. \tag{19}$$

For higher spin nonets, since the corresponding states with equal isospin and alternating parity lie on the linear Regge trajectories, one has, e.g.,

$$
\begin{aligned}
m_{a_2}^2 &= m_\rho^2 + 1/\alpha' &= 2mB + 3/2\alpha', \\
m_{K_2^*}^2 &= m_{K^*}^2 + 1/\alpha' &= (m + m_s)B + 3/2\alpha', \\
m_{f_2}^2 &= m_\omega^2 + 1/\alpha' &\simeq 2mB + 3/2\alpha', \\
m_{f_2'}^2 &= m_\phi^2 + 1/\alpha' &\simeq 2m_sB + 3/2\alpha',
\end{aligned}
\tag{20}
$$

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[2]Also, $m_{D^*}^2 - m_D^2 = 0.55 \text{ GeV}^2$, $m_{D_s^*}^2 - m_{D_s}^2 = 0.58 \text{ GeV}^2$, $m_{B^*}^2 - m_B^2 = 0.55 \text{ GeV}^2$, with the exception of the $c\bar{c}$ states for which $m_{J/\psi}^2 - m_{\eta_c}^2 = 0.70 \text{ GeV}^2$.

$$
\begin{aligned}
m_{\rho_3}^2 &= m_{a_2}^2 + 1/\alpha' = 2mB + 5/2\alpha', \\
m_{K_3^*}^2 &= m_{K_2^*}^2 + 1/\alpha' = (m+m_s)B + 5/2\alpha', \\
m_{\omega_3}^2 &= m_{f_2}^2 + 1/\alpha' \simeq 2mB + 5/2\alpha', \\
m_{\phi_3}^2 &= m_{f_2'}^2 + 1/\alpha' \simeq 2m_sB + 5/2\alpha', \quad \text{etc.,}
\end{aligned}
\tag{21}
$$

and also

$$
\begin{aligned}
m_{b_1}^2 &= m_\pi^2 + 1/\alpha' = 2mB + 1/\alpha', \\
m_{K_1}^2 &= m_K^2 + 1/\alpha' = (m+m_s)B + 1/\alpha', \quad \text{etc.}
\end{aligned}
\tag{22}
$$

Thus, we consider the relations (14) as granted by both the Gell-Mann–Okubo mass formula for a close-to-ideally mixed nonet and the Regge phenomenology. It then follows from (13),(14) that

$$
\begin{aligned}
m\frac{\partial m_{0''}}{\partial m} &\simeq m\frac{\partial m_1}{\partial m} = \frac{mB}{m_1} = \frac{m_\pi^2}{2m_1}, \\
m\frac{\partial m_{1/2}}{\partial m} &= \frac{mB}{2m_{1/2}} = \frac{m_\pi^2}{4m_{1/2}}, \\
m\frac{\partial m_{0'}}{\partial m} &\simeq 0,
\end{aligned}
\tag{23}
$$

and

$$
\begin{aligned}
m_s\frac{\partial m_{0''}}{\partial m_s} &\simeq m_s\frac{\partial m_1}{\partial m_s} = 0, \\
m_s\frac{\partial m_{1/2}}{\partial m_s} &= \frac{m_s B}{2m_{1/2}} = \frac{m_s m_K^2}{2m_{1/2}(m+m_s)} \cong \frac{m_K^2}{2m_{1/2}}, \\
m_s\frac{\partial m_{0'}}{\partial m_s} &\simeq \frac{m_s B}{m_{0'}} = \frac{m_s m_K^2}{m_{0'}(m+m_s)} \cong \frac{m_K^2}{m_{0'}},
\end{aligned}
\tag{24}
$$

since $m_s >> m$. Thus, we find that the expressions to be inserted into Eqs. (10) and (12) are, respectively,

$$
m\frac{\partial m_i}{\partial m} = \frac{m_\pi^2}{2m_i}, \quad m_s\frac{\partial m_i}{\partial m_s} = \frac{m_K^2}{m_i},
\tag{25}
$$

and out of 9 isospin degrees of freedom of a nonet, 6 contribute to the formula for the non-strange condensate: 3 isovector, 2 isospinor and 1 isoscalar which belongs to a mostly singlet, and 3 to the formula for the strange condensate: 2 isospinor and 1 isoscalar which belongs a mostly octet.[3] We shall make another simplification in Eqs. (10),(12), viz., approximate the Bose-Einstein statistics by the Maxwell-Boltzmann

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[3]In fact, each of the $\bar{u}u, \bar{d}d, \bar{s}s$ condensates gains the contribution of 3 isospin degrees of freedom of a nonet.

one, taking account of the sum over $r$ through the factor $\pi^2/6 = \zeta(2) \equiv \sum_r 1/r^2$, which is the asymptotic form of this sum for $T >> m_i$. Then, the formulas (10) and (12) will finally reduce to

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle_0 \left( 1 - \frac{T^2}{24 f_\pi^2} \sum_i' g_i \frac{m_i}{T} K_1 \left( \frac{m_i}{T} \right) \right), \tag{26}$$

$$\langle \bar{s}s \rangle_T = \langle \bar{s}s \rangle_0 \left( 1 - \frac{T^2}{12 f_K^2} \sum_i'' g_i \frac{m_i}{T} K_1 \left( \frac{m_i}{T} \right) \right), \tag{27}$$

where the primes indicate the summation over 6 and 3 isospin degrees of freedom of a nonet, respectively. Now it is seen in Eq. (26) that if one restricts himself to the massless pions alone, $g_\pi = 3$, one obtains the formula (2). This formula sets the temperature scale for the non-strange condensate $\sim 250$ MeV. As seen in Eqs. (26),(27), the massive states accelerate the melting of the condensates, so that the temperature scale for, at least, the non-strange condensate is expected to be much narrower.

To calculate the sums in Eqs. (26),(27), we resort to the notion of a *resonance spectrum* which is introduced in order to substitute the summation over individual particle species by the integration over the mass in the expressions for thermodynamic quantities, so that, e.g., Eqs. (3),(4) may be rewritten (in the Maxwell-Boltzmann approximation for the particle statistics) as

$$p = \int_{m_1}^{m_2} dm \, \tau(m) p(m), \quad p(m) \equiv \frac{m^2 T^2}{2\pi^2} K_2 \left( \frac{m}{T} \right), \tag{28}$$

$$\rho = \int_{m_1}^{m_2} dm \, \tau(m) \rho(m), \quad \rho(m) \equiv T \frac{dp(m)}{dT} - p(m), \tag{29}$$

and the resonance spectrum $\tau(m)$ is normalized as

$$\int_{m_1}^{m_2} dm \, \tau(m) = \sum_i g_i, \tag{30}$$

where $m_1$ and $m_2$ are the masses of the lightest and heaviest species, respectively, entering the formulas (3),(4).

In both the statistical bootstrap model [18, 19] and the dual resonance model [20], a resonance spectrum takes on the form

$$\tau(m) \sim m^a \, e^{m/T_0}, \tag{31}$$

where $a$ and $T_0$ are constants. The treatment of a hadronic resonance gas by means of the spectrum (31) leads to a singularity in the thermodynamic functions at $T = T_0$ [18, 19] and, in particular, to an infinite number of the effective degrees of freedom in

7

the hadron phase, thus hindering a transition to the quark-gluon phase. Moreover, as shown by Fowler and Weiner [21], an exponential mass spectrum of the form (31) is incompatible with the existence of the quark-gluon phase: in order that a phase transition from the hadron phase to the quark-gluon phase be possible, the hadronic spectrum cannot grow with $m$ faster than a power.

In our previous work [22] we considered a model for a transition from a phase of strongly interacting hadron constituents, described by a manifestly covariant relativistic statistical mechanics which turned out to be a reliable framework in the description of realistic physical systems [23], to the hadron phase described by a resonance spectrum, Eqs. (28),(29). An example of such a transition may be a relativistic high temperature Bose-Einstein condensation studied by the authors in ref. [24], which corresponds, in the way suggested by Haber and Weldon [25], to spontaneous flavor symmetry breakdown, $SU(3)_F \rightarrow SU(2)_I \times U(1)_Y$, upon which hadronic multiplets are formed, with the masses obeying the Gell-Mann–Okubo formulas [14]

$$m^\ell = a + bY + c\left[\frac{Y^2}{4} - I(I+1)\right];\qquad(32)$$

here $I$ and $Y$ are the isospin and hypercharge, respectively, $\ell$ is 2 for mesons and 1 bor baryons, and $a, b, c$ are independent of $I$ and $Y$ but, in general, depend on $(p, q)$, where $(p, q)$ is any irreducible representation of $SU(3)$. Then only the assumption on the overall degeneracy being conserved during the transition is required to lead to the unique form of a resonance spectrum in the hadron phase:

$$\tau(m) = Cm, \quad C = \text{const.}\qquad(33)$$

Zhirov and Shuryak [26] have found the same result on phenomenological grounds. As shown in ref. [26], the spectrum (33), used in the formulas (28),(29) (with the upper limit of integration infinity), leads to the equation of state $p, \rho \sim T^6$, $p = \rho/5$, called by Shuryak the "realistic" equation of state for hot hadronic matter [7], which has some experimental support. Zhirov and Shuryak [26] have calculated the velocity of sound, $c_s^2 \equiv dp/d\rho = c_s^2(T)$, with $p$ and $\rho$ defined in Eqs. (3),(4), and found that $c_s^2(T)$ at first increases with $T$ very quickly and then saturates at the value of $c_s^2 \simeq 1/3$ if only the pions are taken into account, and at $c_s^2 \simeq 1/5$ if resonances up to $M \sim 1.7$ GeV are included.

We have checked the coincidence of the results given by the linear spectrum (33) with those obtained directly from Eq. (3) for the actual hadronic species with the corresponding degeneracies, for all well-established hadronic multiplets, both mesonic and baryonic, and found it excellent [22]. Therefore, the theoreticalconclusion that a linear spectrum is the actual spectrum in the description of individual hadronic multiplets finds its experimental confirmation as well. In our recent papers [15, 16] we have shown that a linear spectrum of an individual nonet is consistent with the Gell-Mann–Okubo mass formula (16) (in fact, this formula may be derived with the

help of a linear spectrum [15]), and leads to an extra relation for the masses of the isoscalar states, Eq. (17), which was checked in ref. [16] and shown to hold with an accuracy of up to $\sim 3\%$ for all well-established nonets. In ref. [15] we have generalized a linear spectrum to the case of four quark flavors and derived the corresponding Gell-Mann–Okubo mass formula for an $SU(4)$ meson hexadecuplet, in good agreement with the experimentally established masses of the charmed mesons. In ref. [27] we have applied a linear spectrum to the problem of establishing the correct $q\bar{q}$ assignment for the problematic meson nonets, like the scalar, axial-vector and tensor ones, and separating out non-$q\bar{q}$ mesons. In this paper we shall apply a resonance spectrum to the derivation of the formulas for the temperature dependences of the quark condensates.

It was shown in ref. [22] that the actual resonance spectrum does not depend on the dimensionality of spacetime.[4] Therefore, the sums in Eqs. (26),(27) which are, in fact, related to the expression for the pressure of a free gas in 1+1 dimensions [29], as calculated for individual nonets, may be substituted by the integration over $m$ with a linear spectrum. The normalization constant $C$ was established in ref. [22]: for a nonet, one has 9 isospin degrees of freedom lying in the interval $(m_{0''} \simeq m_1,\ m_{0'})$. Therefore, Eq. (30) gives

$$C \int_{m_{0''}}^{m_{0'}} dm\ m = 9,$$

and hence

$$C = \frac{18}{m_{0'}^2 - m_{0''}^2} \equiv \frac{18}{\triangle} \simeq 27\ \text{GeV}^{-2}, \tag{34}$$

where the difference $\triangle \equiv m_{0'}^2 - m_{0''}^2$ is determined by a distance between the parallel Regge trajectories for the $\omega$ and $\phi$ resonances, which are described by the straight lines $J = 0.59 + 0.84M^2$ and $J = 0.04 + 0.84M^2$, respectively, so that $\triangle \simeq 0.65\ \text{GeV}^2$. We note further that the meson nonets may be arranged in the pairs of nonets which have equal parity but different spins (which differ by 2), e.g.,[5]

$1\ ^3P_0\ J^{PC} = 0^{++}$, $a_0(1320)$, $f_0(1300)$, $f_0(1525)$, $K_0^*(1430)$,
$1\ ^3P_2\ J^{PC} = 2^{++}$, $a_2(1320)$, $f_2(1270)$, $f_2'(1525)$, $K_2^*(1430)$,

$1\ ^3D_1\ J^{PC} = 1^{--}$, $\rho(1700)$, $\omega(1600)$, $K^*(1680)$, (no $\phi$ candidate),
$1\ ^3D_3\ J^{PC} = 3^{--}$, $\rho_3(1700)$, $\omega_3(1600)$, $\phi_3(1850)$, $K_3^*(1780)$,

$3\ ^1S_0\ J^{PC} = 0^{-+}$, $\pi(1770)$, $\eta(1760)$, $K(1830)$, (no $\eta'$ candidate),
$1\ ^1D_2\ J^{PC} = 2^{-+}$, $\pi_2(1670)$, $K_2(1770)$, (no $\eta, \eta'$ candidates),
and occupy the mass interval of an individual nonet but have 18 isospin degrees of

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[4]This is another argument against the Hagedorn spectrum, since the exponent $a$ in Eq. (31) depends explicitly on the dimensionality of spacetime (it is related to the number of transverse dimensions of a string theory [28]).

[5]For the scalar meson nonet, we use the $q\bar{q}$ assignment suggested by the authors in ref. [16, 27].

freedom in this interval, i.e., twice as much as that for an individual nonet. Moreover, as the temperature gets closer to the critical one of chiral symmetry restoration, we expect the chiral partners (the states with equal isospin but different parity) have equal masses and form parity doublets. The work of DeTar and Kogut [30] shows convincingly that the "screening masses" of chiral partners are different below and become equal above a common $T_c$. This work was carried out for four quark flavors. Similar results were obtained for two flavors by Gottlieb *et al.* [31]. In these calculations, the chiral partners were $(\pi, \sigma)$, $(\rho, a_1)$ and $(N(\frac{1}{2}+), N(\frac{1}{2}-))$. Thus, we expect the correct density of states per unit mass interval to be twice as much as that for an individual nonet, and hence, the correct normalization constant is

$$C \simeq 54 \text{ GeV}^{-2}. \tag{35}$$

In the case we are considering here, one has 6 isospin degrees of freedom in the interval $m_{1/2}^2 - m_{0''}^2 = \triangle/2$, in view of (17), contributing to the formula for the non-strange condensate, and 3 in the interval $m_{0'}^2 - m_{1/2}^2 = \triangle/2$ contributing to the formula for the strange condensate; i.e., a lower half of a linear spectrum of a nonet contributes to $\langle \bar{q}q \rangle_T$, while an upper half to $\langle \bar{s}s \rangle_T$. The corresponding normalization constants are

$$C_{\bar{q}q} = \frac{12}{\triangle/2} = \frac{24}{\triangle} = \frac{4}{3}C \simeq 70 \text{ GeV}^{-2}, \tag{36}$$

$$C_{\bar{s}s} = \frac{6}{\triangle/2} = \frac{12}{\triangle} = \frac{2}{3}C \simeq 35 \text{ GeV}^{-2}. \tag{37}$$

Once the mass spectrum of a nonet (with a given fixed spin) is established to be linear, one may take into account different nonets with different spins in Eqs. (3),(4). As shown in ref. [22], since the particle spin is related to its mass, $J_i \sim \alpha' m_i^2$, $\alpha'$ being a universal Regge slope, the spin degeneracy turns out to be proportional to the mass squared, and the account for different nonets results in the following mass spectrum,

$$\tau'(m) = C' m^3, \quad C' = 2\alpha' C \simeq 90 \text{ GeV}^{-4}, \tag{38}$$

which is the actual resonance spectrum of hadronic matter and leads to the equation of state [34]

$$p, \rho \sim T^8, \quad p = \rho/7. \tag{39}$$

Bebie *et al.* [9] have calculated the ratio $\rho/p$ directly from Eqs. (3),(4), with all known hadron resonances with the masses up to 2 GeV taken into account, and found that the curve $\rho/p$ first decreases very quickly and then saturates at the value of $\rho/p \simeq 7$, as read off from Fig. 1 of ref. [9], in agreement with (39).

In order to show that the obtained normalization constant is correct, we note that the number of states with the masses up to $M$, given by the mass spectrum (38), is

$$N(M) = \frac{C' M^4}{4} \simeq 22.5 \, (M, \text{ GeV})^4. \tag{40}$$

For, e.g., $M = 1.25$ Eq. (40) gives

$$N(1.25) \simeq 55. \tag{41}$$

The masses up to 1.25 GeV have the members of the pseudoscalar and vector meson nonets, and the $h_1(1170)$, $b_1(1235)$ and $a_1(1260)$ mesons, the mass of the latter was indicated by the recent Particle Data Group as 1.23 GeV [32]. We do not include the scalar mesons $a_0(980)$ and $f_0(980)$ which seem to be non-$q\bar{q}$ objects [27], but may include the $f_0(1300)$ meson which has the mass lying in the interval $1 - 1.5$ GeV, according to the recent Particle Data Group. Thus, we have 9+27+1+9+9+1=56 actual mesonic species having the masses up to 1.25 GeV, in excellent agreement with (41).

For $M = 1.7$ GeV, Eq. (40) gives

$$N(1.7) \simeq 188. \tag{42}$$

As seen in the Meson Summary Table [33], the masses up to 1.7 GeV have the members of the following nonets: $1\,^1S_0$, $1\,^3S_1$, $1\,^1P_1$, $1\,^3P_0$, $1\,^3P_1$, $1\,^3P_2$, $2\,^1S_0$, $2\,^3S_1$. Therefore, one has

$$(20 \text{ spin states}) \times (9 \text{ isospin states}) = 180 \text{ states}, \tag{43}$$

in good agreement with the result (42) given by a cubic spectrum.

For $M = 2$ GeV, Eq. (40) gives

$$N(2) \simeq 360. \tag{44}$$

The masses up to 2 GeV have the members of all the nonets indicated in [33] except for the $1\,^3F_4$ and $2\,^3P_2$ nonets. In this case, one has

$$(41 \text{ spin states}) \times (9 \text{ isospin states}) = 369 \text{ states}, \tag{45}$$

again in good agreement with the result (44) given by a cubic spectrum. Thus, we consider the cubic spectrum (38) as granted by the actual experimental meson spectrum.

Similarly to the case of an individual nonet, one finds the normalization constants

$$C'_{\bar{q}q} = 2\alpha' C_{\bar{q}q} \simeq 120 \text{ GeV}^{-4}, \quad C'_{\bar{s}s} = 2\alpha' C_{\bar{s}s} \simeq 60 \text{ GeV}^{-4}, \tag{46}$$

and the relations (10),(12) finally take on the forms, respectively,

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle_0 \left( 1 - \frac{T^2}{8f_\pi^2} - \frac{120 \text{ GeV}^{-4} T^2}{24 f_\pi^2} \int dm \, m^3 \frac{m}{T} K_1 \left( \frac{m}{T} \right) \right), \tag{47}$$

$$\langle \bar{s}s \rangle_T = \langle \bar{s}s \rangle_0 \left( 1 - \frac{60 \text{ GeV}^{-4} T^2}{12 f_K^2} \int dm \, m^3 \frac{m}{T} K_1 \left( \frac{m}{T} \right) \right), \tag{48}$$

11

where we have separated out the contribution of the pions which may well be treated as massless at temperatures $\sim 150$ MeV, and taken into account the remaining hadronic species by the resonance spectrum (38) with the normalization constants given in (46). We have found that the contribution of the kaons (as well as $\eta$ and $\eta'$), if separated out as well in Eqs. (47),(48), is comletely negligible at these temperatures. We include these states together with the other mesonic states in the integrals of Eqs. (47),(48), which, therefore, have the lower limit of integration $\sim 0.5$ GeV. With respect to the remaining integration in (47),(48), we note that the main contribution to integrals of this type is given by the mass region in which $m \gg T$; therefore, one may extend the upper limit of integration to infinity and neglect the lower limit, and obtain, through the formula [35]

$$\int_0^\infty dx\, x^\mu K_\nu(ax) = 2^{\mu-1} a^{-\mu-1} \Gamma\left(\frac{1+\mu+\nu}{2}\right) \Gamma\left(\frac{1+\mu-\nu}{2}\right),$$

$$\langle \bar{q}q \rangle_T \simeq \langle \bar{q}q \rangle_0 \left[ 1 - \left(\frac{T}{260 \text{ MeV}}\right)^2 - \left(\frac{T}{220 \text{ MeV}}\right)^6 \right], \tag{49}$$

$$\langle \bar{s}s \rangle_T \simeq \langle \bar{s}s \rangle_0 \left[ 1 - \left(\frac{T}{235 \text{ MeV}}\right)^6 \right]. \tag{50}$$

Temperature dependences of the condensates, as given by (49),(50), are shown in Fig. 1. The critical temperature of the chiral restoration transition is $T_c \simeq 190$ MeV, in agrement with the result of Gerber and Leutwyler [4] obtained for the pions and massive states with the nonzero masses of $u$- and $d$-quarks. For the strange quark condensate, the temperature at which it is melted out is $\simeq 235$ MeV, as seen directly in (50). For $T \simeq 190$ MeV, Eq. (50) gives $\langle \bar{s}s \rangle_{190} \simeq 0.72\, \langle \bar{s}s \rangle_0$, i.e., only 28% of the strange quark condensate melts by the chiral restoration transition. It is seen in Eq. (49) that the share of the pions in the overall reduction of the non-strange condensate from its initial value at $T = 0$ to zero at $T_c$ is about 53%, the remaining 47% is the contribution of the massive states. We note also that Eq. (49) may be well approximated by

$$\langle \bar{q}q \rangle_T \simeq \langle \bar{q}q \rangle_0 \left[ 1 - \left(\frac{T}{190 \text{ MeV}}\right)^3 \right]. \tag{51}$$

The formulas (49),(50) have been obtained for the meson resonances alone. There is no difficulty of principle to consider the baryon resonances in a similar way, with the inclusion of two chemical potentials, for both conserved net baryon number and strangeness. As we have checked in ref. [22], a mass spectrum of the $SU(3)$ baryon multiplets is linear, as well as for the meson nonets, although to establish its correspondence to the Gell-Mann–Okubo formulas is more difficult than for a meson nonet, since these formulas are linear in mass for baryons (more detailed discussion is given in ref. [22]). Recent result of Kutasov and Seiberg [36] shows that the numbers of

bosonic and fermionic states in a non-supersymmetric tachyon-free string theory must approach each other as increasingly massive states are included. The experimental hadronic mass spectrum shows that in the mass range $\sim 1.2 - 1.7$ GeV, the number of baryon states nearly keeps pace with that of meson states [37] (and, therefore, is well described by the same cubic spectrum as for the mesons, Eq. (38)). Above $\sim 1.7$ GeV, the number of the observed baryons begins to outstrip that of the mesons, and then greatly surpasses the latter at higher energies (indicating, therefore, that the baryon resonance spectrum grows faster than (38) in this mass region, since the cubic spectrum (38) describes the meson resonances well, up to, at least, 2 GeV, as we have seen in Eqs. (41)-(45)). The explanation of this behavior of the experimental resonance spectrum was found by Cudell and Dienes in a naive hadron-scale string picture [38]: the ratio of the numbers of the baryon and meson states should, in fact, oscillate around unity, with the mesons favored first, then baryons, then mesons again, etc. Keeping in mind this picture, we may assume that the "in-average" baryon resonance spectrum has the same form, Eq. (38), as the meson resonance one. It is then possible to estimate the contribution of the baryon resonances to the temperature dependences of the quark condensates by assuming that the formulas (26),(27) hold for the baryon resonances, as well as for meson ones, and that out of 27 degrees of freedom of $SU(3)$ baryon octet, nonet and decuplet, 2/3, i.e., 18, contribute to $\langle \bar{q}q \rangle$, and 1/3, i.e., 9, to $\langle \bar{s}s \rangle$. If one now neglects, for simplicity, the baryon number and strangeness chemical potentials (i.e., considers the case of both zero net baryon number and strangeness), and takes into account the baryon resonances along with the meson ones by the mass spectrum (38), one will obtain the same formulas, (47),(48), but with the factors in front of integrals which are twice as much as those in the case of the meson resonances alone. These formulas will further reduce to the relations

$$\langle \bar{q}q \rangle_T \simeq \langle \bar{q}q \rangle_0 \left[ 1 - \left( \frac{T}{260 \text{ MeV}} \right)^2 - \left( \frac{T}{195 \text{ MeV}} \right)^6 \right], \tag{52}$$

$$\langle \bar{s}s \rangle_T \simeq \langle \bar{s}s \rangle_0 \left[ 1 - \left( \frac{T}{210 \text{ MeV}} \right)^6 \right], \tag{53}$$

shown in Fig. 2. One sees that now the critical temperature of the chiral symmetry restoration transition is $T_c \simeq 175$ MeV, while the strange quark condensate melts out at $\simeq 210$ MeV, and about 34% of the latter melts by the chiral restoration transition. Now the pions' contribution to the overall reduction of the value of the non-strange condensate is about 45%, the remaining 55% is the share of the massive states.

## Concluding remarks

We have derived the formulas for the temperature dependences of the quark condensates by taking into account the contribution of the massive states parametrized by a

resonance spectrum which has been established by the authors in previous papers. In the case of the meson resonances alone, our results agree those obtained previously by Gerber and Leutwyler, and suggest the chiral symmetry restoration transition to occur at the critical temperature $T_c \simeq 190$ MeV. With the baryon resonances taking into account along with the meson ones, the critical temperature turns out to be $T_c \simeq 175$ MeV, which is closer to the currently adopted value $140 - 150$ MeV established by lattice gauge calculations [39]. In either case, only $\sim 30\%$ of the strange quark condensate melts by the chiral restoration transition, in agreement with lattice calculations [5]. The share of the pions in the overall reduction of the non-strange condensate from its initial value at $T = 0$ to zero at $T_c$ is $\sim 50\%$ in either case, the remaining $\sim 50\%$ is the contribution of the massive states.

# Acknowledgements

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FIGURE CAPTIONS

Fig. 1. The ratio $\langle\bar{q}q\rangle_T/\langle\bar{q}q\rangle_0$ as a function of temperature for a) non-strange, b) strange quark condensate, in the case of the meson resonances alone, as given in Eqs. (49),(50).

Fig. 2. The same as Fig. 1 but in the case of both the meson and baryon resonances, as given in Eqs. (52),(53).